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FORMATION OF A JET IN CASE OF NON-STATIONARY FLOW OF A PERFECT FLUID FROM A SLIT*

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The problem of flow of a perfect fluid from a slit separating two parallel planes is given a mathematical formulation for the case of a submerged jet, as well as for the case of a flow with a free boundary. Various types of flow are classified. The results of numerical solutions are compared with experimental data. The phenomenon of reversal of the vortex sheet which occurs when the flow rate through the slit is reduced, is discussed. The self-modelling problem is solving using the method of matching the asymptotic expansions. A cumulative effect is discovered, namely that the rate of penetration of a narrow central part of the self-modelling jet exceeds the rate of flow of the main part of the jet by one order of magnitude.

1. We assume that the rate of flow Q of fluid through the slit and the slit width $2l$, are both sufficiently arbitrary functions of time t. To find the unqiue solution we must demand that the Chaplygin—Joukovskii condition that the velocity is finite at the sharp edges of the slit $x = \pm l(t)$, $y = 0$ holds. Since the fluid particles acquire vertical motion after passing the slit edges, a line of tangential discontinuity of velocity will emerge from the edges. This will be the vortex sheet, forming the front of the jet. We assume that such generalized solutions of the Euler equations with break in the values of the hydrodynamic functions at the vortex sheet represent the limiting form of the laminar flow of viscous fluid at large Reynold's numbers.

The flow pattern depends essentially on the initial data. We shall consider a motion from the state at rest. If at the initial instant $t = 0$ the slit is closed, i.e. $l(0) = 0$, then a flow pattern is possible in which all fluid particles forming the front of the stream pass through the slit and acquire vorticity (Fig.1). If $l(0) \neq 0$, then the particles forming at $t = 0$ a line of contact between the fluids situated in the upper and lower half-plane will remain, in accordance with the Lagrange theorem, uncurved (dashed line in Fig.2); the jet front will have a mushroom shape, with the free ends of the vortex coiled into two spirals (solid lines in Fig.2, $t > 0$).

The number of dimensions of the problem can be reduced using the method of integral boundary equations. This replaces the two-dimensional Laplace equation for the velocity potential with the corresponding conditions at the solid boundary and at the jet front at which the potential becomes discontinuous, by a one-dimensional integrodifferential equation of evolution of the vortex sheet. Since the flow is symmetric about the y-axis, the left half of the vortex sheet will be a mirror image of the right half, and we can write equation of the latter in the parametric form as $z = z(\Gamma, t)$ where $z = x + iy$ and Γ is the circulation varying along the portion of the vortex sheet in question and counted from its free end. The function

$$2z/l = \zeta^{1/2} + \zeta^{-1/2} \tag{1.1}$$

maps the region of flow into the outside of the semi-infinite segment $\operatorname{Im}\zeta = 0, 0 \leqslant \operatorname{Re}\zeta \leqslant \infty$. Figs.2 and 3 show the correspondence of the points. Making use of the symmetry of the problem, we conclude that the velocity of the fluid on both sides of the cut in the ζ-plane is the same, i.e. there are no attached vortices. Consequently, it is in this plane that the influence of the solid boundaries of the flow can be accounted for in the simplest manner.

In order to arrange the flow at the points $D(\zeta = 0)$ and $C(\zeta = \infty)$ corresponding to the points at infinity in the physical plane, we must arrange a point source of strength $Q(t)$ and a point sink of the same strength. Thus the solution of the problem depends on the actual

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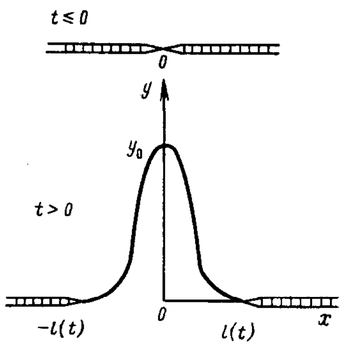

Fig.1

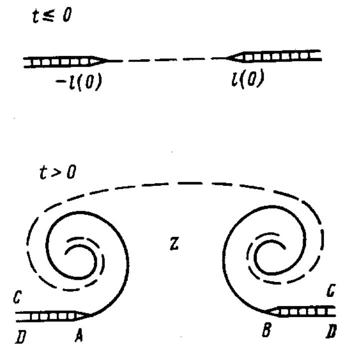

Fig.2

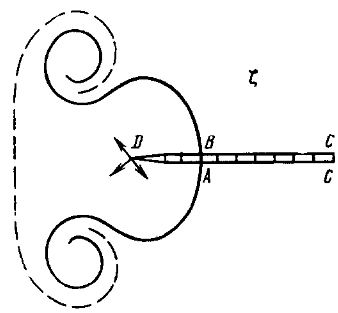

Fig.3

form of the defining functions $Q(t)$ and $l(t)$. It appears that the formation of vortical spiral structures with an infinite number of turns at some instant of time t_0 is connected with the nonanaliticity of these functions at the point $t = t_0$. The complex flow potential $W(\zeta, t)$ is a function piecewise continuous at the vortex sheet and determined by the existence of a point source and two symmetric segments of the vortex sheet

$$W(\zeta, t) = \frac{Q}{2\pi} \ln \zeta - \frac{1}{2\pi i} \int_0^{\Gamma_0} \ln \frac{\zeta - \zeta(\Gamma, t)}{\zeta - \overline{\zeta(\Gamma, t)}}\, d\Gamma \tag{1.2}$$

The singular integrals along the segments of the vortex sheet are Cauchy type integrals, the upper bar denotes a complex conjugate and Γ_0 denotes the complete circulation of a single segment of the vortex sheet. The Chaplygin – Joukovskii condition is given at the point $\zeta = 1$ and has the form $\partial W(1, t)/\partial\zeta = 0$, or

$$Q = i \int_0^{\Gamma_0} \frac{\overline{\zeta(\Gamma, t)} - \zeta(\Gamma, t)}{|1 - \zeta(\Gamma, t)|^2}\, d\Gamma \tag{1.3}$$

Two modes of flow are possible: a flow into a submerged half-space when the motion of fluid on both sides of the stream front must be considered, and a flow with a free boundary simulating e.g. a penetration of a stream of water into air.

2. First we consider the problem of a submerged jet. The densities and total pressures of both fluids appearing at the initial instant in the upper and lower half-space shall be assumed equal. The equation of evolution of the vortex sheet (see e.g. /1/) has the form

$$\frac{\partial \overline{z(\Gamma, t)}}{\partial t} = \frac{4}{l} \frac{\zeta^{3/2}}{\zeta - 1} \left[\frac{Q}{2\pi\zeta} - \frac{1}{2\pi i} \int_0^{\Gamma_0} \frac{d\Gamma'}{\zeta - \zeta(\Gamma', t)} + \frac{1}{2\pi i} \int_0^{\Gamma_0} \frac{d\Gamma'}{\zeta - \overline{\zeta(\Gamma', t)}} \right] \tag{2.1}$$

and the relation connecting $z(\Gamma, t)$ and $\zeta(\Gamma, t)$ is given by (1.1). Let us now formulate, for the nonlinear singular equation (2.1) with condition (1.3), a conditionally correct problem with initial conditions in the usual manner. The solution is sought in the class of piecewise analytic functions satisfying the Hölder condition; the contour of the vortex sheet is assumed simple and smooth. Numerical calculations are carried out using the method of discrete vortices /2/, with help of the linear regularizer /3/ successfully used in solving the problem of detached flow past plane bodies and wings with small aspect ratio.

Numerical computations have shown that the spiral nucleus of the vortex sheet increases in size with time. In the case of a constant flow rate the part of the vortex sheet situated near the edges stabilizes and approaches the form corresponding to a stationary jet flow. When the flow rate decreases over some period of time, a vorticity of opposite sign may appear, which can displace the part of the vortex sheet adjacent to the edge, into the lower half-space. The appearance of such reversed vortex sheet rapidly disrupts the flow in the course of actual experiment, and in the numerical computations.

Fig.4 depicts the dependence of the dimensionless rate of variation of complete circulation of a single spiral segment of the vortex sheet $d(\Gamma_0/Q_0)/d\tau$ on time, obtained by numerical methods for the particular case when the flow of fluid through the slit of constant width l varies according to the periodic law $Q = Q_0(2 + 0.5\sin 20\tau)$ where $\tau = tQ_0/l^2$. After some transition process, $d\Gamma_0/d\tau$ emerges into a periodic mode.

A flow of fluid through a slit in the working part of the water pipe was studied experimentally under various modes of opening the throttle controlling the water flow $Q(t)$. The flow was made observable by dyeing the outflowing liquid, or by feeding a dye to the slit edges. The number $R = Q/\nu$ where ν is the kinematic viscosity coefficient, was equal to $2\cdot 10^3$. The experimental data were used to obtain the relation connecting the dimensionless depth of penetration of the jet y_0/l with the dimensionless time τ, for the constant flow rate and $l =$ const. Numerical computation for these conditions gave the linear relation $y_0/l = 2.7\tau$. The agreement between the experimental and numerical results is satisfactory, and the deviation from linearity insignificant. The velocity at the points C and D at infinity is zero, therefore the pressure at these points (in the flow with a free boundary the pressure at the point C is constant) is equal to

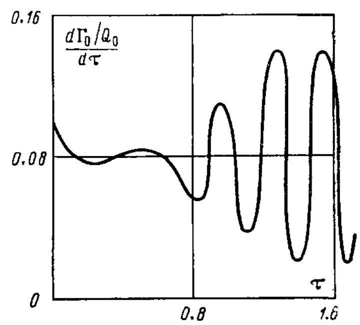

Fig.4

$$p = p_0 - \rho\,\mathrm{Re}\left(\frac{\partial w}{\partial t} - \frac{2\zeta}{l}\frac{\zeta+1}{\zeta-1}\frac{dl}{dt}\frac{\partial w}{\partial \zeta}\right)$$

where ρ is the fluid density. Using the equations (1.2) for the complex velocity potential, we obtain the limiting values for the pressure at the points $C\,(\zeta\to\infty)$ and $D\,(\zeta\to 0)$:

$$p_C = p_0 - \frac{\rho}{2\pi}\frac{dQ}{dt}\ln|\zeta| + \frac{\rho Q}{\pi l}\frac{dl}{dt}$$

$$p_D = p_0 - \frac{\rho}{2\pi}\frac{dQ}{dt}\ln|\zeta| - \frac{\rho Q}{\pi l}\frac{dl}{dt} + \frac{2\rho\Gamma_0}{\pi i l}\frac{dl}{dt}\int_0^1 \frac{\zeta(\lambda,t) - \overline{\zeta(\lambda,t)}}{|1-\zeta(\lambda,t)|^2}\,d\lambda + p_1(t)$$

$$\lambda = \frac{\Gamma}{\Gamma_0}, \quad p_1(t) = \frac{\rho}{\pi}\frac{d\Gamma_0}{dt}\int_0^1 \arg\zeta(\lambda,t)\,d\lambda$$

At $dQ/dt \neq 0$ the pressure is infinite. In the case of the attached flow, i.e. when the condition (1.3) is absent and $\Gamma_0 \equiv 0$, widening of the slit leads to lowering of the pressure at the point D, and increasing it at C by the same amount. Narrowing the slit produces the opposite result. In the case of a detached flow we transform the expression for p_D using the condition (1.3)

$$p_D = p_0 - \frac{\rho}{2\pi}\frac{dQ}{dt}\ln|\zeta| + \frac{\rho Q}{\pi l}\frac{dl}{dt} + p_1(t)$$

When $dQ/dt = 0$, then the pressure drop $p_D - p_C$ is finite and equal to $p_1(t)$. When $d\Gamma_0/dt > 0$, we have $p_D > p_C$.

3. If the fluid flow rate is sufficiently low and the rate of widening the slit sufficiently high, then the vortical front of the jet in the scheme depicted in Fig.1 will differ little from the segment $y = 0, -l(t) \leqslant x \leqslant l(t)$. In this case, or more accurately in the absence of formation of spiral vortices, $(|\partial y/\partial x| < \infty)$ and the linear theory can be used when

$$|Q| \ll |dl^2/dt| \tag{3.1}$$

Let us expand the solution in a power series in small parameter $\varepsilon = \max(Qdt/dl^2)$

$$z(\Gamma, t;\ \varepsilon) = x_0(g,t) + \varepsilon z_1(g,t) + O(\varepsilon^2), \quad z_1 = x_1 + iy_1$$
$$Q(t) = \varepsilon Q_1(t),\ \Gamma = \varepsilon g + O(\varepsilon^2),\ \Gamma_0 = \varepsilon\Gamma_1 + O(\varepsilon^2) \tag{3.2}$$

From (1.1) it follows that the form of the vortex sheet in the ζ-plane will differ little from the circle $|\zeta| = 1$

$$\zeta(g, t;\ \varepsilon) = e^{i\theta} + \varepsilon\zeta_1(g,t) + O(\varepsilon^2) \tag{3.3}$$
$$\theta = 2\arccos\frac{x_0}{l}, \quad \frac{z_1}{l} = \frac{i}{2}\zeta_1 e^{-i\theta}\sin\frac{\theta}{2}$$

We substitute the principal terms of the expansions (3.2) and (3.3) into (1.3) and (2.1). The Chaplygin–Joukovskii condition (1.3) determines the functional relationship between the total circulation

$\Gamma_1(t)$ and rate of flow $Q_1(t)$:

$$Q_1(t) = \int_0^{\Gamma_1(t)} \operatorname{ctg}\frac{\theta}{2}\, dg \tag{3.4}$$

Terms appearing in the right-hand side of the equation (2.1) are of the order of ε, therefore the right-hand side can be neglected in determining $x_0(g, t)$, and we obtain

$$x_0 = x_0(g) \tag{3.5}$$

The solution (3.5) has a simple physical meaning. The displacement of the vortical points (i.e. of the points for which $\Gamma = \text{const}$) occurs only in the vertical direction since their abscissa in time independent. The equation of evolution (2.1) yields an ordinary differential equation for y_1 characterizing the vertical displacement of the vortex sheet, the right-hand side of which is a known function of g and t, provided that the relationship between $x_0(g)$ and $\Gamma_1(t)$ is known. A solution of this equation with boundary condition $y_1(\Gamma_1, t) = 0$ will be discussed below, using a concrete example of a self-modelling flow.

We note that if the abscissas of the vortical points were known at the initial instant $t = 0$ and situated within the range $-x_0[\Gamma_1(0)] \leqslant x_0(g) \leqslant x_0[\Gamma_1(0)]$, then the solution (3.5) establishes the same distribution of the circulation over the x-axis for these points at the later instants. In other words, the position of the vortical points on the x-axis is stationary. However, "generation" of vorticity at the sharp edges and the widening of the slit cause the appearance of nonstationarity, new vortex points are added to the existing points, and according to (3.5) the horizontal position of these points should be stationary. The relation connecting the circulation $g = \Gamma_1$ of the generated vortex points and the time t_1 of their appearance is obtained from the equation $\Gamma_1 = \Gamma_1(t)$, which is assumed known. The converse dependence of time on the circulation $t_1 = t_1(\Gamma_1)$ is also assumed known. Since the abscissas of the vortices generated at the edges are equal to $\pm l(t_1)$, the solution (3.5) for a vortex sheet appearing at $t > 0$ assumes the form

$$x_0 = l[t_1(g)] \tag{3.6}$$

4. In the case of a monotonously opening slit the solution of the self-similar problem is of interest. Let the slit width increase according to the power law $l(t) = kt^n$, $k > 0$. The outflow of fluid is self-similar if the flow rate Q is proportional to t^{2n-1}. We shall consider a real case of $n > 0.5$ when the flow of fluid is finite at the initial instant. We introduce the dimensionless variables according to the formulas

$$z(\Gamma, t) = kt^n\mu(\lambda),\quad \Gamma = 2\pi nk^2t^{2n-1}G,\quad \Gamma_0 = 2\pi nk^2t^{2n-1}G_0 \tag{4.1}$$
$$Q = 2\pi nk^2t^{2n-1}q,\quad \lambda = G/G_0,\quad 0 < m = 2 - 1/n < 2$$

Passing in (2.1) and (1.3) to the dimensionless variables, we obtain

$$\bar{\mu} - m\lambda\frac{d\bar{\mu}}{d\lambda} = \frac{4\zeta^{3/2}}{\zeta - 1}\left[\frac{q}{\zeta} + iG_0\int_0^1\frac{d\lambda'}{\zeta(\lambda) - \zeta(\lambda')} - iG_0\int_0^1\frac{d\lambda'}{\zeta(\lambda) - \zeta(\bar{\lambda}')}\right] \tag{4.2}$$

$$q = iG_0\int_0^1\frac{\overline{\zeta(\lambda)} - \zeta(\lambda)}{|1 - \zeta(\lambda)|^2}\, d\lambda \tag{4.3}$$

The condition of applicability of the linear theory (3.1) holds for $q \ll 1$. Following (3.2) and (3.3), we expand the solution into a power series in terms of the small parameter q:

$$\mu(\lambda; q) = \alpha_0(\lambda) + q\mu_1(\lambda) + O(q^2),\quad \mu_1 = \alpha_1 + i\beta_1 \tag{4.4}$$
$$G_0 = qG_1 + O(q^2),\quad \zeta(\lambda, q) = e^{i\theta_0} + q\zeta_1(\lambda) + O(q^2)$$
$$\theta_0 = 2\arccos\alpha_0,\quad \mu_1 = i/2\zeta_1e^{-i\theta_0}\sin\theta_0/2$$

In the self-similar variables the solution (3.5) has the form

$$\alpha_0 = \lambda^{1/m} \tag{4.5}$$

The circulation of the velocity G_1 is found from the condition (4.3)

$$G_1^{-1}(m) = m\int_0^1\alpha_0^{\,m}\frac{d\alpha_0}{\sqrt{1 - \alpha_0^2}} \tag{4.6}$$

Equating the terms of the order of q in (4.2) to zero, we obtain an ordinary linear differential equation for determining the ordinates of the vortex sheet $\beta_1(\lambda)$:

$$\left(\beta_1 - m\lambda \frac{d\beta_1}{d\lambda}\right)\sin\frac{\theta_0}{2} = 2 + G_1 I(\lambda, m), \quad I(\lambda, m) = 2\int_0^1 \frac{\sin\theta_0(\lambda')\,d\lambda'}{\cos\theta_0(\lambda') - \cos\theta_0(\lambda)}, \quad \theta_0(\lambda) = 2\arccos\lambda^{1/m} \tag{4.7}$$

with the boundary condition $\beta_1(1) = 0$.

In the particular case of uniform widening of the diaphragm $(m = 1)$ the integrals in the right-hand part of (4.6) and (4.7) are calculated in quadratures, and the solution of (4.7) is represented in this case by the function

$$\beta_1(\lambda) = \lambda \int_0^1 \frac{d\lambda}{\lambda^2} \ln\frac{1 - \sqrt{1-\lambda^2}}{1 + \sqrt{1-\lambda^2}}$$

which has the following logarithmic singularity as $\lambda \to 0$:

$$\beta_1(\lambda \to 0) \to -2\ln\lambda \tag{4.8}$$

When $\lambda \to 0$, the integral $I(\lambda, m)$ is finite if $m > 1$ and has a power singularity if $m < 1$. In the latter case $(m < 1)$ equation (4.7) yields

$$\beta_1(\lambda \to 0) \to \text{const } \lambda^{1-1/m} \tag{4.9}$$

Thus the velocity at the center of the self-similar jet calculated according to the linear theory becomes infinite at $m \leqslant 1$. We investigate this paradox of the linear theory of perfect fluid using the method of matching asymptotic expansions.

When $m \leqslant 1$, the linear theory (outer expansion) becomes unsuitable in some q^r-neighborhood of the point $\lambda = 0$ where $r > 0$. The formulas (4.8) and (4.9) determine the inner limit of the outer expansion for $m = 1$ and $m < 1$ respectively. The value of r and the order of magnitude of $\mu(\lambda)$ over the inner region are found from the condition of matching the inner limit with the outer limit, and from the estimation of the circulation $\lambda: r = 2n - 1, \mu \sim q^n$. Consequently the inner expansion has the form

$$\mu(\lambda; q) = q^n \sigma(\lambda_0) + o(q^n), \quad \lambda = q^{2n-1}\lambda_0 \tag{4.10}$$

Substituting the expansion (4.10) into (4.2), we obtain the equation of evolution of the central part of the self-similar jet for $m \leqslant 1$

$$\overline{\sigma(\lambda_0)} - m\lambda_0 \frac{\overline{d\sigma(\lambda_0)}}{d\lambda_0} = -2i\delta_{1,m} + iG_1 \int_0^\infty \left[\frac{1}{\sigma(\lambda_0) - \sigma(\lambda_0')} - \frac{1}{\sigma(\lambda_0) - \overline{\sigma(\lambda_0')}}\right] d\lambda_0', \quad \delta_{1,m} = \begin{cases} 1, & m = 1 \\ 0, & m \neq 1 \end{cases} \tag{4.11}$$

Solid boundaries do not appear in the inner problem, therefore the Chaplygin–Joukovskii condition becomes meaningless and must be replaced by the condition

$$\operatorname{Im}\sigma(\lambda_0 = \infty) = 0 \tag{4.12}$$

which ensures the matching between the inner and outer expansion.

It can be shown that (4.11) has no unbounded solutions when $\lambda_0 \to 0$. Indeed, in the presence of an unbounded solution the right-hand part of (4.11) will tend to a constant. But then the differential operator appearing in the left-hand side of the equation will only have bounded solutions which contradicts the initial assumption. Hence the velocity at the center of the jet is finite. Thus when $n \leqslant 1$ and the rate of flow q is sufficiently small, the self-similar jet exerts a cumulative effect: the rate of penetration of its narrow central part $(x \sim q^n)$ where a small portion of the circulation $(\lambda \sim q^{2n-1})$ is concentrated, has the order of q^n and exceeds by one order of magnitude the velocity of the main body of the jet which is of the order of q.

5. Let us inspect the mode of flow with a free boundary. In this case the pressure p along the discontinuity is the same and does not change with time. The component of the velocity v normal to the free boundary is equal to the rate of displacement of the jet front

$$v = \left(\frac{\partial y}{\partial t}\right)_x \left[1 + \left(\frac{\partial y}{\partial x}\right)^2\right]^{-1/2} \tag{5.1}$$

where $y = y(x, t)$ describes the form of the free boundary. The tangential velocity component u determines the strength of the vortex sheet.

To derive the equation of evolution, we introduce the following parameter along the free boundary:

$$h = \varphi - \int \frac{p_0(t) - p}{\rho} dt$$

where φ is the velocity potential at the free boundary. Let us inspect the rate of variation of this parameter $\delta h/\delta t$ at the vortex point, i.e. at the point situated at the free boundary and moving along this boundary at the rate of $u/2$ (in contrast to a vortex point at the vortex sheet, such a point does not correspond to the value $h = \text{const}$). From the Bernoulli equation

$$\frac{\delta\varphi}{\delta t} - \frac{v^2}{2} = \frac{p_0 - p}{\rho}$$

we find

$$\frac{\delta h}{\delta t} = \frac{\delta\varphi}{\delta t} - \frac{p_0 - p}{\rho} = \frac{v^2}{2}$$

The complex velocity of the point in question is equal to

$$\frac{\delta\bar{z}}{\delta t} = \frac{\partial\bar{z}}{\partial t} + \frac{\partial\bar{z}}{\partial h}\frac{\delta h}{\delta t} = \overline{\frac{\partial z(h, t)}{\partial t}} + \frac{v^2}{2}\overline{\frac{\partial z(h, t)}{\partial h}}$$

In accordance with the Sokhotskii formulas, the equation of evolution for the free boundary is obtained by equating this velocity with the complex rate of flow $\partial W/\partial z$:

$$\frac{\partial\bar{z}}{\partial t} + \frac{1}{2}\left(\frac{\partial y}{\partial t}\right)_x^2 \left[1 + \left(\frac{\partial y}{\partial x}\right)^2\right]^{-1} \frac{\partial\bar{z}}{\partial h} = \frac{4}{l}\frac{\zeta^{3/2}}{\zeta - 1}\left[\frac{Q}{2\pi\zeta} - \frac{1}{2\pi i}\int_0^{h_0} \frac{dh'}{\zeta - \zeta(h', t)} + \frac{1}{2\pi i}\int_0^{h_0} \frac{dh'}{\zeta - \overline{\zeta(h', t)}}\right] \tag{5.2}$$

The equation of evolution for the free boundary of the jet (5.2) differs from (2.1) by the presence in its left-hand side of the second term proportional to the square of the velocity component v normal to the discontinuity and given by (5.1). In the case of a self-similar flow, the variables are reduced to their dimensionless form with help of the formulas (4.1) where h is formally replaced by Γ. The equation of evolution (5.2) assumes the form

$$\bar{\mu} - mG\frac{d\bar{\mu}}{dG} + \frac{1}{4\pi}\frac{(\beta - \alpha\beta')^2}{1 + (\beta')^2}\frac{d\bar{\mu}}{dG} = \frac{4\zeta^{3/2}}{\zeta - 1}\left[\frac{q}{\zeta} + iG_0\int_0^1 \frac{d\lambda'}{\zeta - \zeta(\lambda')} - iG_0\int_0^1 \frac{d\lambda'}{\zeta - \overline{\zeta(\lambda')}}\right] \tag{5.3}$$

$$\mu = \alpha + i\beta, \quad \beta' = d\beta/d\alpha$$

and the Chaplygin—Zhukovskii condition (4.3) remains in force.

The outer solution of (5.3) coincides, for $q \ll 1$, in the first approximation, with the solution of (4.2), the latter equation describing the outflow of a self-similar jet into a submerged space, since the order of the additional term in (5.3) is greater than the order of the velocity in the linear theory. In the inner expansion the orders of those two quantities are equal. We have

$$\bar{\sigma} - m\lambda_0\frac{d\bar{\sigma}}{d\lambda_0} + \frac{1}{4\pi G_1}\frac{(\beta - \alpha\beta')^2}{1 + (\beta')^2}\frac{d\bar{\sigma}}{d\lambda_0} = -2i\delta_{1,m} + iG_1\int_0^\infty \left[\frac{1}{\sigma(\lambda_0) - \sigma(\lambda_0')} - \frac{1}{\sigma(\lambda_0) - \overline{\sigma(\lambda_0')}}\right] d\lambda_0' \tag{5.4}$$

and condition (4.12) holds.

Just as for the equation (4.11), we can show that the solution is bounded as $\lambda \to 0$. It follows therefore that the cumulative effect takes place also in the self-similar flow of fluid with a free boundary.

Apart from the problem of jet automatics, the above problem can also be used to study three-dimensional stationary gas flows of which the above problem represents a two-dimensional analog. Indeed, if the narrow zone of separation is stretched along the direction of the unperturbed stream, then the longitudinal variable in the equations of gas dynamics degenerates and a law of plane flows becomes valid, in the first approximation to which the compressibility of the medium is neglected. The theory of wing with small aspect ratio, nonlinear theory of wing with finite aspect ratio in a supersonic /4/ and subsonic /5/ flow, and a flow with local vortex sheet past bodies with arbitrary aspect ratio /6/ are the known examples of this. In this sense the plane nonstationary flows of fluid through a slit are equivalent to the detached flow past an arbitrary cut in a screen. An attached flow past an arbitrary cut in a wing was discussed in /7/. A similar problem (with periodically distributed slits) arises in the course of dealing with the problem of filtration of gas through perforated boundaries with arbitrary slits, provided that the number $R = Q/\nu$ is sufficiently large. The nonviscous mechanism of appearance of resistance to the filtration of gas, is connected with the formation of vortex sheets moving away from the slit edges. Adoption of such an approach to the problem obviates the necessity of formulating the empirical boundary conditions at the perforated walls.

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